

A STUDY ON CALCULATION OF OPTIMUM GEAR RATIOS OF A TWO-STAGE HELICAL GEARBOX WITH SECOND STAGE DOUBLE GEAR SETS

NGUYEN THI HONG CAM¹, VU NGOC PI¹, TRAN THI HONG²,
LE HONG KY³ & LUU ANH TUNG¹

¹Thai Nguyen University of Technology, Thai Nguyen, Vietnam

²Nguyen Tat Thanh University, Ho Chi Minh city, Vietnam

³Vinh Long University of Technology Education, Vinh Long, Vietnam

ABSTRACT

The article focuses on finding optimum calculation of the gear ratios of a two-stage helical gearbox with second stage double gear sets. To obtain the optimum gear ratios, an optimization problem was established. Besides, the gearbox length was chosen as the objective function of the problem. In addition, attention was paid on to the influences of the input parameters such as the total gearbox ratio, the wheel face width coefficient, the allowable contact stress and the output torque. To explore the effects of these factors on the optimum gear ratios, a simulation experiment and then a computer program were constructed. The findings reported some models for obtaining the optimum gearbox ratios accurately in a simple way.

KEYWORDS: Gear Ratio, Optimum Gear Ratio, Optimum Gearbox Design & Helical Gearbox

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1. INTRODUCTION

Gear ratios play a significant role in optimum gearbox design since they have large effects on the dimension, mass and cost of a gearbox. That is the reason why numerous researchers have paid great attention to gear ratio determination. V. N. Kudreavtev et al. [1] introduced the graph method used for calculating the gear ratio of the first stage of a two-stage helical gearbox (see Figure 1). According to this method, to determine the gear ratio u_1 , coefficient λc^3 must be selected in the range 0.6 – 4, which reveals the complication and, more importantly, the optimum values cannot be obtained.

G. Milou et al. [2] reported another method called practical method. In this method, the gear ratios were determined based on practical data gathered from gearbox factories. It was mentioned that the weight of a two-stage gearbox was minimum, if the ratio of two centres distances a_{w2} / a_{w1} received values from 1.4 to 1.6 [2]. Besides, the optimum gear ratios were presented in the tabulated form.

The most commonly used method for determining the optimum gear ratios in practice has been the model method. To apply the method, it is required to construct the optimization problem of which, the objectives can be minimum volume of gears [3], the minimum cross section dimension of the gearbox [4], the minimum gearbox mass [5] or minimum mass of gears [6]. Based on the findings, the optimum gear ratio model is obtained.

Regarding two-stage helical gearbox with second stage double gear sets, the model method was employed

to find the optimum gear to achieve the minimum cross section [7] and the minimum gearbox length [8]. Nevertheless, the effects of the input parameters on the optimum gears were not assessed.

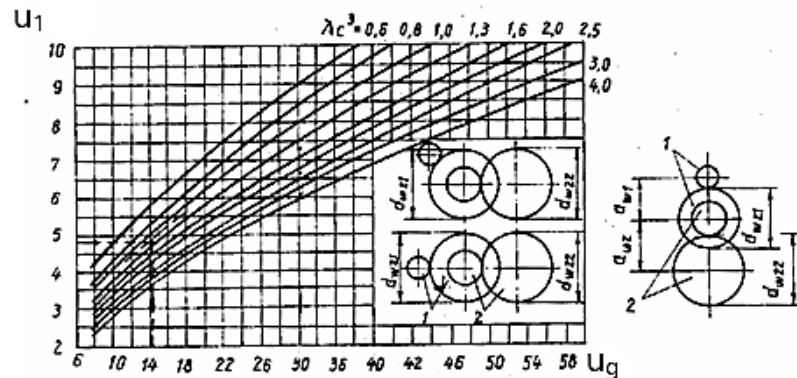


Figure 1: The Gear Ratio of the Stage 1 Versus the Total Gearbox Ratio [1]

This paper presents a study aiming at finding the optimum gear ratios of a two-stage helical gearbox with second stage double gear sets. In the research, an optimization problem with the objective as the minimum gearbox length was solved. In addition, the impacts of the input parameters on the optimum gear ratios were examined.

2. OPTIMIZATION PROBLEM

For a two stage helical gearbox with second stage double gear sets, the gearbox length is found by (Figure 2):

$$L = d_{w11} / 2 + a_{w1} + a_{w2} + d_{w22} / 2 \quad (1)$$

Where, a_{w1} and a_{w2} are the center distances of the first and the second stages; d_{w11} and d_{w22} are the pitch diameters (mm) of the first and the second stages which d_{w22} can be calculated as [9]:

$$d_{w11} = 2 \cdot a_{w1} / (u_1 + 1) \quad (2)$$

$$d_{w22} = 2 \cdot a_{w2} \cdot u_2 / (u_2 + 1) \quad (3)$$

In addition, the gear ratio of the second stage is determined by:

$$u_2 = u_g / u_1 \quad (4)$$

Where, u_1 , u_2 are the gear ratios of the first and the second stages; u_g is the total gearbox ratio.

From (1), (2), (3) and (4) the following equation is given:

$$L = f(a_{w1}, a_{w2}, u_1, u_g) \quad (5)$$

Consequently, the optimization problem is defined as:

$$\text{minimize } L \quad (6)$$

With the following constraint:

$$5 \leq u_g \leq 35 \quad (7)$$

It can be found from equations (5) and (6) that to solve the optimization problem, it is necessary to determine the centre distances of the first stage a_{w1} and the second stage a_{w2} .

Based on equations (5) and (6), it is noticed that the center distances of the first stage a_{w1} and the second stage a_{w2} are required for solving the optimization problem.

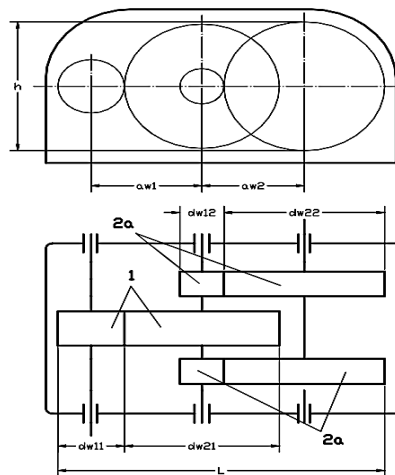


Figure 2: Calculation Schema

2.1 Calculating the Center Distance of the First Stage

For the gearbox, the center distance of the first stage a_{w1} is determined as calculated as in [9]:

$$a_{w1} = K_a \cdot (u_1 + 1) \cdot \left(\frac{T_{11} \cdot k_{H\beta}}{[\sigma_H]^2 \cdot u_1 \cdot \psi_{ba1}} \right)^{1/3} \quad (8)$$

In which, $K_{H\beta}$ is the contact load ratio for pitting resistance; we can choose $k_{H\beta} = 1.1$ because $k_{H\beta} = 1.02 \div 1.28$ [9]; $[\sigma_H]$ is the allowable contact stress (MPa); In practice, $[\sigma_H] = 350 \dots 410$ (MPa); k_a is the material coefficient; for the gear material is steel $k_a = 43$ [9]; ψ_{ba} is the coefficient of wheel face width; for the first stage of the gearbox $\psi_{ba1} = 0.3 \dots 0.35$;

From the moment equilibrium condition of the gearbox, we have:

$$T_{out} = T_{11} \cdot \eta_{hg}^2 \cdot \eta_{be}^3 \cdot u_g \quad (9)$$

In which,

- $K_{H\beta}$ is the contact load ratio for pitting resistance; according to [9], for the first stage of the gearbox

$k_{H\beta} = 1.02 \div 1.28$. Thus, $K_{H\beta}$ can be selected as 1.1;

- $[\sigma_H]$ is the allowable contact stress (MPa); In practice, $[\sigma_H] = 350 \dots 410$ (MPa);
- k_a is the material coefficient; However, the gear material is normally steel, thus $k_a = 43$ [9];
- ψ_{ba} is the coefficient of wheel face width; for the first stage of the gearbox $\psi_{ba1} = 0.3 \dots 0.35$;

Based on the moment equilibrium condition of the gearbox, we have:

$$T_{out} = T_{11} \cdot \eta_{hg}^2 \cdot \eta_{be}^3 \cdot u_g \quad (10)$$

In which, η_{hg} is the helical gear transmission efficiency ($\eta_{hg} = 0.96 \dots 0.98$ [9]); η_{be} is the transmission efficiency of a pair of rolling bearings ($\eta_{be} = 0.99 \dots 0.995$ [9]).

Taking $\eta_{hg} = 0.97$ and $\eta_{be} = 0.992$ and substituting them into (10) gets:

$$T_{11} = 1.0887 \cdot T_{out} / u_g \quad (11)$$

Substituting (11) and $k_{H\beta} = 1.1$ into (8) leads to:

$$a_{w1} = 45.6635 \cdot (u_1 + 1) \cdot \left(\frac{T_{out}}{[\sigma_{H1}]^2 \cdot u_1 \cdot u_g \cdot \psi_{ba1}} \right)^{1/3} \quad (12)$$

2.2 Calculating the Center Distance of the Second Stage

The center distance of the second stage a_{w2} can be determined by [9]:

$$a_{w2} = K_a \cdot (u_2 + 1) \cdot \left(\frac{T_{12} \cdot k_{H\beta}}{[\sigma_H]^2 \cdot u_2 \cdot \psi_{ba2}} \right)^{1/3} \quad (13)$$

Similarly, for the second stage we have:

$$T_{out} = 2 \cdot T_{12} \cdot \eta_{hg}^2 \cdot \eta_{be}^2 \cdot u_2 \quad (14)$$

Choosing $\eta_{hg} = 0.97$ and $\eta_{be} = 0.992$ as in section 2.1 and substituting them into (14) give:

$$T_{12} = 0.5238 \cdot T_{out} / u_2 \quad (15)$$

Substituting (15), $k_a = 43$ and $k_{H\beta} = 1.1$ (as in section 2.1) into (13) gets:

$$a_{w2} = 35.7812 \cdot (u_2 + 1) \cdot \left(\frac{T_{out}}{[\sigma_H]^2 \cdot u_2^2 \cdot \psi_{ba2}} \right)^{1/3} \quad (16)$$

2.3 Experimental Work

Table 1: Input Parameters

Factor	Code	Unit	Low	High
Total gearbox ratio	u_g	-	5	35
Coefficient of wheel face width of stage 1	x_{ba1}	-	0.3	0.35
Coefficient of wheel face width of stage 2	x_{ba2}	-	0.35	0.4
Allowable contact stress of stage 1	AS_1	MPa	350	420
Allowable contact stress of stage 2	AS_2	MPa	350	420
Output torque	T_{out}	Nmm	10^5	10^7

As mentioned above, the influences of the input parameters on the optimum gear ratios needed to be investigated. Thus, a simulation experiment was constructed, and regarding the design, a 2-level full factorial one was selected. In the next step, 6 input factors for the investigation were put into consideration (see Table 1). As a result, the number of tests required to be implemented consist of 64. To carry out the experiment, a computer program was created based on equations (6) and (7). The input parameters at different levels and the program output (the optimum gear ratio of the second stage u_2) are reported in Table 2.

3. OPTIMIZATION RESULTS AND DISCUSSIONS

Figure 3 presents a graph of the major influence of each input parameter, which is used to assess the effects of the input factors on the response and their relative strength. It is recognized from the Figure that the optimum gear ratio u_2 are remarkably contingent on the total gearbox ratio u_g . It is directly proportional to the total gearbox ratio. Moreover, it is dependent upon the coefficient of the wheel face width of the first and the second stages (ψ_{ba1} and ψ_{ba2}) and the allowable contact stress of the first and the second stages AS_1 and AS_2 . Nevertheless, the optimum gear ratio is not influenced by the output torque T_{out} .

The standardized effects are presented from the largest to the smallest value in the Pareto chart (see Figure 4). As it can be seen from the graph, those factors including the total gearbox ratio (factor A), the allowable contact stress of the first and the second stages AS_1 and AS_2 (factors D and E), the coefficients of wheel face width of the first and the second stages (factors B and C) and the interactions between them (AB and AC) are represented by the bars which cross the reference line. This indicates that the input factors are statistically significant at the 0.05 level with the response model (the gear ratio of the second stage u_2). Contrarily, the other factors including the allowable contact stress of the helical gear set (factor D) and the output torque (factor E) symbolized by the bars that do not cross the reference line, which reveals that these factors (D and E) do not have effects on the response model.

Table 2: Experimental Plans and Output Response

Std Order	Run Order	Center Pt	Blocks	u_g	X_{ba1}	X_{ba2}	AS_1 (MPa)	AS_2 (MPa)	T_{out} (Nm)	u_2
57	1	1	1	5	0.3	0.35	420	420	10000	2.26
44	2	1	1	35	0.35	0.35	420	350	10000	4.25
39	3	1	1	5	0.35	0.4	350	350	10000	2.25

27	4	1	1	5	0.35	0.35	420	420	100	2.22
Table 2 Contd.,										
45	5	1	1	5	0.3	0.4	420	350	10000	2.19
14	6	1	1	35	0.3	0.4	420	350	100	4.54
...										
63	63	1	1	5	0.35	0.4	420	420	10000	2.25
58	64	1	1	35	0.3	0.35	420	420	10000	4.79

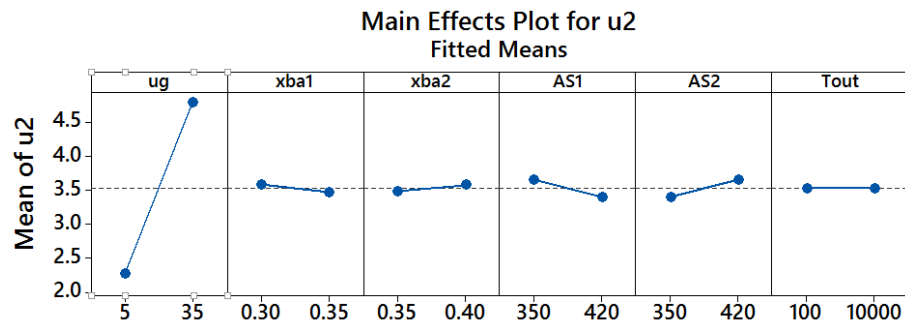


Figure 3: Main Effects Plot for the Optimum Gear Ratio of the Second Stage

The standardized effects on the response are demonstrated in Figure 5. As displayed in the graph, the most significant factor for the optimum gear ratio is the total gearbox ratio (factor A). Additionally, those factors including the allowable contact stress, the coefficients of the wheel face width and the interactions AE, AC, CE and BD have positive standardized effects. Specifically, if these factors alter from low to high values, the optimum gear ratio of the first stage rises. On the contrary, the allowable contact stress and the coefficient of wheel face width of the first stage have a negative standardized effect. When they increase, the optimum gear ratio declines.

The estimated effects and coefficients for the optimum gear ratio u_2 are reported in Figure 6. It is noticed that the total gearbox ratio u_g , the coefficient of the wheel face width (ψ_{ba1} and ψ_{ba2}) and the allowable contact stress of the first and the second stages AS_1 AS_2 , and their interactions are significant to the optimum gear ratio as their P-value is lower than 0.05. The following equation illustrates the relation between the optimum gear ratio and those factors:

$$\begin{aligned}
 u_2 = & 0.829 + 0.08271u_g + 0.650 xba1 + 1.283 xba2 + 0.001137AS_1 + 0.002399 AS_2 - 0.08500 u_g * xba_1 \\
 & + 0.07833 u_g * xba_2 - 0.000145 u_g * AS_1 + 0.000145 u_g * AS_2 - 3.00 xba_1 * xba_2 + 0.002857 xba_1 * AS_1 - \\
 & 0.002857 xba_1 * AS_2 - 0.002857 xba_2 * AS_1 + 0.002857 xba_2 * AS_2 - 0.000005 AS_1 * AS_2
 \end{aligned} \quad (17)$$

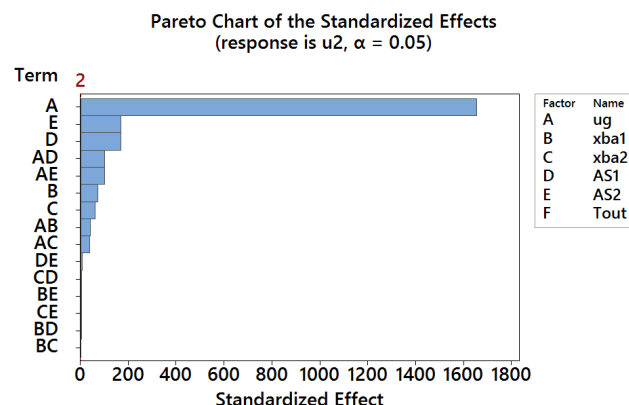


Figure 4: Pareto Chart of the Standardized Effects

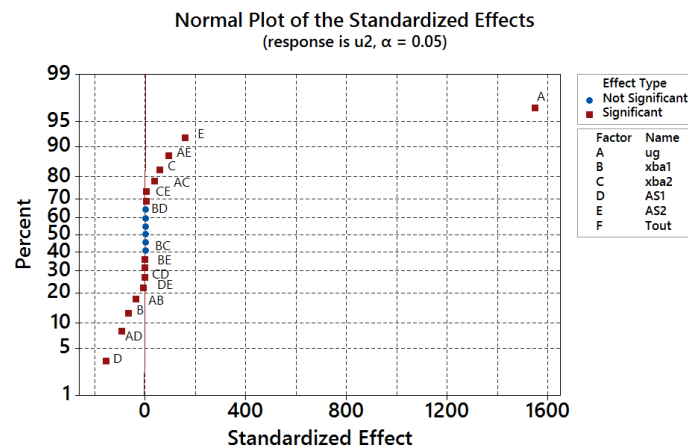


Figure 5: Normal Plot for the Optimum Gear Ratio of the Second Stage

It is recognized that the adj-R2 and pred-R2 possess high values (Figure 6), which proves that the mentioned model fits the data very well. Furthermore, the optimum gear ratio of the second stage u_2 can be determined by using Equation (16). Based on u_2 , the gear ratio of the first stage can be calculated by $u_1 = u_g / u_2$.

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		3.52562	0.00077	4605.86	0.000	
ug	2.53375	1.26687	0.00077	1655.04	0.000	1.00
xba1	-0.108750	-0.054375	0.000765	-71.04	0.000	1.00
xba2	0.093750	0.046875	0.000765	61.24	0.000	1.00
AS1	-0.257500	-0.128750	0.000765	-168.20	0.000	1.00
AS2	0.257500	0.128750	0.000765	168.20	0.000	1.00
ug*xba1	-0.063750	-0.031875	0.000765	-41.64	0.000	1.00
ug*xba2	0.058750	0.029375	0.000765	38.38	0.000	1.00
ug*AS1	-0.152500	-0.076250	0.000765	-99.61	0.000	1.00
ug*AS2	0.152500	0.076250	0.000765	99.61	0.000	1.00
xba1*xba2	-0.003750	-0.001875	0.000765	-2.45	0.018	1.00
xba1*AS1	0.005000	0.002500	0.000765	3.27	0.002	1.00
xba1*AS2	-0.005000	-0.002500	0.000765	-3.27	0.002	1.00
xba2*AS1	-0.005000	-0.002500	0.000765	-3.27	0.002	1.00
xba2*AS2	0.005000	0.002500	0.000765	3.27	0.002	1.00
AS1*AS2	-0.011250	-0.005625	0.000765	-7.35	0.000	1.00

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0061237	100.00%	100.00%	100.00%

Figure 6: Estimated Effects and Coefficients for the Optimum Gear Ratio of the Second Stage

4. CONCLUSIONS

In this study, an optimization problem for calculation of the gear ratios of a two-stage helical gearbox with second stage double gear sets was solved. The findings revealed that the minimum length of a two-stage helical gearbox with second stage double gear sets can be obtained based on the optimum gear ratios of the gearbox. To identify the optimum gear ratios, some models were built. It was found that the optimum gear ratios of the gearbox could be determined accurately and simply, as the estimated models are explicit.

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